Additional Mathematics

GCE 'O' Levels

Mock Examination · Total: 94 marks · Generated 30/08/2026

Question 1
[6 marks]

POLYNOMIALS AND PARTIAL FRACTIONS

3 A cuboid with volume $(2x+1)^2$ cm^3 has a rectangular base with dimensions x^2 cm and $(2x-1)$ cm. Find an expression for the height of the cuboid, leaving your answer in partial fractions. [6]

Question 2
[8 marks]

TRIGONOMETRIC FUNCTIONS AND IDENTITIES

11 (a) Prove that $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = \tan x$. [4]

(b) Hence, solve the equation $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = 5 - 2\sec^2 x$ for $0^\circ < x < 360^\circ$. [4]

Question 3
[7 marks]

PROOFS IN PLANE GEOMETRY

3 In the diagram, PQS lies on a circle and $PCBQ$ lies on another circle. PQ is a common chord of the two circles. BPS is a straight line. QC is a tangent to the circle PQS at Q. QC and BP intersect at M .

(a) Prove that BC is parallel to QS. [3]

(b) Prove that triangle PQM is similar to triangle QSM. [2]

(c) Hence show that $QM^2 = PM \cdot SM$. [2]

Question 4
[10 marks]

QUADRATIC FUNCTIONS

12 (a) (i) Show that $x^2 - 10x + 21$ can never be smaller than -4.

(ii) Given that $y = x^2 - 10x + 21$, write down the coordinates of the turning point and state whether the point is a maximum or minimum.

(iii) Solve the equation $x^2 - 10x + 21 = 0$.

(iv) Hence, sketch the graph of $y = x^2 - 10x + 21$.

(b) Given that $6x^2 + \dfrac{x}{a} - \dfrac{1}{b^2} = 0$, where a, b ⊟ 0, show that there are no values of a and b for which the equation has equal roots.

Question 5

[12 marks]

COORDINATE GEOMETRY

9 A circle, C_1, has equation

$$x^2 + y^2 + 8x - 10y = 40$$

(i) Find the radius and the coordinates of the centre of C_1. [3]

A second circle, C_2, has a diameter PQ. The point P has coordinates $(0, 6)$ and the equation of the tangent to C_2 at Q is

$$4y = 3x - 26$$

(ii) Find the equation of the line PQ and hence the coordinates of Q. [4]

(iii) Find the radius and the coordinates of the centre of C_2. [3]

(iv) Explain why the point $(2, 6)$ lies within both the circles C_1 and C_2. [2]

Question 6

[11 marks]

EQUATIONS AND INEQUALITIES

(a) Find the range of values of k for which the curve $y = kx^2 + (k + 2)x + 3$ lies entirely above the x-axis. [5]

(b) For the case $k = 2$, determine whether the line $y = 4x + 1$ intersects the curve, and if so find the coordinates of any point(s) of intersection. [6]

Question 7

[9 marks]

BINOMIAL EXPANSIONS

7 (a) By considering the general term in the binomial expansion of a given expression (the expression appears as an image in the source and is not legible),

(i) explain why there are no odd powers of x in this expansion; [3]

(ii) explain why the term independent of x does not exist. [2]

(b) (i) Find, in descending powers of x, the first 3 terms in the expansion of a given expression (not legible in the source). [2]

(ii) Hence find the value of a given numerical expression, giving your answer correct to 2 decimal places. [2]

Question 8

[11 marks]

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

7 (a) Solve the equation $3^x(18 + 3^x) = 40$. [3]

(b) Solve the equation $\log_2 y^2 = \log_2 3 + \log_2(y + 6)$. [5]

(c) In order to obtain a graphical solution of the equation

$$x = 2\ln\left(4 - \frac{3}{2}x\right),$$

a suitable straight line can be drawn on the same set of axes as the graph of $y = 4 - e^{x/2}$. Make $e^{x/2}$ the subject of $x = 2\ln\left(4 - \frac{3}{2}x\right)$ and hence find the equation of this line. [3]

Question 9

[8 marks]

SURDS

Do not use a calculator in this question.

(a) Solve the equation $\sqrt{3x + 7} - \sqrt{x + 1} = 2$. [4]

(b) Solve the equation $\sqrt{5x + 1} = 1 + \sqrt{3x}$. [4]

Question 10

[12 marks]

DIFFERENTIATION AND INTEGRATION

The diagram shows part of the curve $y = \dfrac{10}{12 - x} - 1$ passing through the point $P(2k, k - 1)$, where k is a constant. The curve meets the x-axis at the point X. The tangent and normal at P meet the x-axis at the points T and N respectively.

(a) Find the equation of the normal at P. [6]

(b) Find the exact area of the shaded region. [6]

Answers

Question 1

Volume = base area × height, so

$$\text{height} = \frac{(2x+1)^2}{x^2(2x-1)} = \frac{4x^2+4x+1}{x^2(2x-1)}$$

Let

$$\frac{4x^2+4x+1}{x^2(2x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1}$$

Multiplying by $x^2(2x-1)$:

$$4x^2+4x+1 = Ax(2x-1) + B(2x-1) + Cx^2$$

Let $x = 0$: $1 = -B$, so $B = -1$.

Let $x = \frac{1}{2}$: $1+2+1 = \frac{C}{4}$, so $C = 16$.

Comparing coefficients of x^2: $4 = 2A + C = 2A + 16$, so $A = -6$.

Check the coefficient of x: $-A + 2B = 6 - 2 = 4$, as required.

Therefore

$$\text{height} = \frac{16}{2x-1} - \frac{6}{x} - \frac{1}{x^2} \text{ cm}$$

Question 2

(a)

Using $\sin 2x = 2\sin x\cos x$, $\cos 2x = 1 - 2\sin^2 x$ in the numerator and $\cos 2x = 2\cos^2 x - 1$ in the denominator:

$$\text{LHS} = \frac{2\sin x\cos x - (1 - 2\sin^2 x) + 1}{2\sin x\cos x + (2\cos^2 x - 1) + 1}$$

$$= \frac{2\sin x\cos x + 2\sin^2 x}{2\sin x\cos x + 2\cos^2 x} = \frac{2\sin x(\cos x + \sin x)}{2\cos x(\sin x + \cos x)}$$

$$= \frac{\sin x}{\cos x} = \tan x = \text{RHS}$$

(b)

Using part **(a)** and $\sec^2 x = 1 + \tan^2 x$:

$$\tan x = 5 - 2\left(1 + \tan^2 x\right)$$

$$2\tan^2 x + \tan x - 3 = 0 \quad \Rightarrow \quad (2\tan x + 3)(\tan x - 1) = 0$$

$$\tan x = 1 \quad \Rightarrow \quad x = 45^\circ,\ 225^\circ$$

$$\tan x = -\frac{3}{2} \quad \Rightarrow \quad \text{basic angle } 56.31^\circ,\ x = 123.7^\circ,\ 303.7^\circ$$

$$x = 45^\circ,\ 123.7^\circ,\ 225^\circ,\ 303.7^\circ$$

Question 3

3(a)

Since QC is tangent to circle PQS at Q, by the tangent-chord (alternate segment) theorem,

$$\angle PQC = \angle PSQ.$$

In circle $PCBQ$, angles subtended by chord PC in the same segment give

$$\angle PQC = \angle PBC.$$

Hence $\angle PBC = \angle PSQ$. These are equal alternate angles for lines BC and QS cut by the transversal BPS, so BC is parallel to QS.

3(b)

By the alternate segment theorem, $\angle PQM = \angle PQC = \angle PSQ = \angle QSM$.

$\angle PMQ = \angle QMS$ (common angle).

Therefore triangle PQM is similar to triangle QSM (AA similarity).

3(c)

Since triangle PQM is similar to triangle QSM, corresponding sides are proportional:

$$\frac{QM}{SM} = \frac{PM}{QM}.$$

Hence $QM^2 = PM \cdot SM$.

Question 4

(a)(i) Completing the square:

$$x^2 - 10x + 21 = (x - 5)^2 - 25 + 21 = (x - 5)^2 - 4.$$

Since $(x - 5)^2 \geq 0$, we have $(x - 5)^2 - 4 \geq -4$, so $x^2 - 10x + 21$ can never be smaller than -4.

(a)(ii) The turning point is $(5, -4)$, and since the coefficient of x^2 is positive it is a minimum.

(a)(iii)

$$x^2 - 10x + 21 = (x - 7)(x - 3) = 0 \Rightarrow x = 7 \text{ or } x = 3.$$

(a)(iv) The graph is an upward parabola with vertex $(5, -4)$, x-intercepts at $(3, 0)$ and $(7, 0)$, and y-intercept $(0, 21)$.

(b) For $6x^2 + \frac{1}{a}x - \frac{1}{b^2} = 0$, the discriminant is

$$\left(\frac{1}{a}\right)^2 - 4(6)\left(-\frac{1}{b^2}\right) = \frac{1}{a^2} + \frac{24}{b^2}.$$

Since $a, b \neq 0$, both $\frac{1}{a^2} > 0$ and $\frac{24}{b^2} > 0$, so the discriminant is strictly positive and can never equal 0. Hence there are no values of a and b for which the equation has equal roots.

Question 5

(i) Complete the square:

$$(x^2 + 8x) + (y^2 - 10y) = 40$$

$$(x + 4)^2 - 16 + (y - 5)^2 - 25 = 40$$

$$(x + 4)^2 + (y - 5)^2 = 81$$

Centre $(-4, 5)$ and radius 9 units.

(ii) The tangent $4y = 3x - 26$ gives $y = \frac{3}{4}x - \frac{13}{2}$, so its gradient is $\frac{3}{4}$. Since PQ is a diameter, PQ is perpendicular to the tangent at Q:

$$m_{PQ} = -\frac{4}{3}$$

Through $P(0, 6)$:

$$y = -\frac{4}{3}x + 6$$

At Q the line meets the tangent:

$$-\frac{4}{3}x + 6 = \frac{3}{4}x - \frac{13}{2}$$

Multiplying by 12:

$$-16x + 72 = 9x - 78$$

$$25x = 150$$

$$x = 6, \qquad y = -\frac{4}{3}(6) + 6 = -2$$

So Q is $(6, -2)$.

(iii) The centre of C_2 is the midpoint of PQ:

$$\left(\frac{0 + 6}{2}, \frac{6 - 2}{2}\right) = (3, 2)$$

The radius is half of PQ:

$$r = \frac{1}{2}\sqrt{(6 - 0)^2 + (-2 - 6)^2} = \frac{1}{2}\sqrt{36 + 64} = \frac{1}{2}(10) = 5$$

Centre $(3, 2)$ and radius 5 units.

(iv) Distance from $(2, 6)$ to the centre of C_1:

$$\sqrt{(2 + 4)^2 + (6 - 5)^2} = \sqrt{37} \approx 6.08$$

Since $6.08 < 9$, the point lies inside C_1.

Distance from $(2, 6)$ to the centre of C_2:

$$\sqrt{(2-3)^2 + (6-2)^2} = \sqrt{17} \approx 4.12$$

Since $4.12 < 5$, the point lies inside C_2. Hence $(2, 6)$ lies within both circles.

Question 6

(a) The curve lies entirely above the x-axis when it opens upwards and has no real roots: $k > 0$ and discriminant < 0.

$$(k+2)^2 - 4k(3) < 0 \Rightarrow k^2 + 4k + 4 - 12k < 0 \Rightarrow k^2 - 8k + 4 < 0.$$

The roots of $k^2 - 8k + 4 = 0$ are $k = 4 \pm 2\sqrt{3}$, so $4 - 2\sqrt{3} < k < 4 + 2\sqrt{3}$. Since $4 - 2\sqrt{3} > 0$, the condition $k > 0$ is automatically met, giving $4 - 2\sqrt{3} < k < 4 + 2\sqrt{3}$.

(b) With $k = 2$ the curve is $y = 2x^2 + 4x + 3$. Setting equal to the line: $2x^2 + 4x + 3 = 4x + 1 \Rightarrow 2x^2 + 2 = 0 \Rightarrow x^2 = -1$, which has no real solution. Hence the line does not intersect the curve.

Question 7

The specific binomial expressions are not legible in the source. The mark scheme records the reasoning and results:

(a)(i) The general term produces powers of x of the form x^{6r} (an even exponent for every non-negative integer r), so no odd powers of x appear.

(a)(ii) A term independent of x would require the exponent of x to be zero, which forces r to be non-integer; since r must be a whole number, no such term exists.

(b)(ii) Substituting the required value into the first three terms gives the approximation 6014.25 (to 2 decimal places).

The underlying expressions are missing, so these results cannot be independently verified.

Question 8

(a) Let $u = 3^x$. Then

$$u(18 + u) = 40 \Rightarrow u^2 + 18u - 40 = 0 \Rightarrow (u + 20)(u - 2) = 0.$$

Since $u = 3^x > 0$, reject $u = -20$; so $3^x = 2$, giving

$$x = \frac{\ln 2}{\ln 3} = 0.631\,(3\text{ s.f.}).$$

(b)

$$\log_2 y^2 = \log_2 3 + \log_2(y + 6) = \log_2 \left[3(y + 6)\right].$$

Comparing arguments, $y^2 = 3(y + 6)$, so

$$y^2 - 3y - 18 = 0 \Rightarrow (y - 6)(y + 3) = 0.$$

Since $y > 0$, reject $y = -3$; hence $y = 6$.

(c) From $x = 2\ln\left(4 - \frac{3}{2}x\right)$:

$$\frac{x}{2} = \ln\left(4 - \frac{3}{2}x\right) \Rightarrow e^{x/2} = 4 - \frac{3}{2}x.$$

Since $y = 4 - e^{x/2}$, we have $e^{x/2} = 4 - y$, so $4 - y = 4 - \frac{3}{2}x$, giving the line

$$y = \frac{3}{2}x.$$

Question 9

(a) Write $\sqrt{3x + 7} = 2 + \sqrt{x + 1}$ and square: $3x + 7 = 4 + 4\sqrt{x + 1} + (x + 1)$, so $2x + 2 = 4\sqrt{x + 1}$, giving $x + 1 = 2\sqrt{x + 1}$. With $u = \sqrt{x + 1}$, $u^2 = 2u$, so $u = 0$ or $u = 2$, i.e. $x = -1$ or $x = 3$. Both satisfy the original equation.

(b) Squaring: $5x + 1 = 1 + 2\sqrt{3x} + 3x$, so $2x = 2\sqrt{3x}$, giving $x = \sqrt{3x}$ and $x^2 = 3x$. Hence $x = 0$ or $x = 3$; both check in the original equation.

Question 10

(a) Since $P(2k, k-1)$ lies on the curve,

$$k - 1 = \frac{10}{12-2k} - 1 \Rightarrow k = \frac{10}{12-2k} \Rightarrow k(12-2k) = 10 \Rightarrow k^2 - 6k + 5 = 0,$$

so $k = 1$ or $k = 5$. From the diagram P lies above the x-axis, so $k = 5$ and $P(10, 4)$.

With $y = 10(12-x)^{-1} - 1$,

$$\frac{dy}{dx} = \frac{10}{(12-x)^2}.$$

At $P(10, 4)$: $\dfrac{dy}{dx} = \dfrac{10}{(12-10)^2} = \dfrac{5}{2}$, so the normal gradient is $-\dfrac{2}{5}$. The normal at P is

$$y - 4 = -\frac{2}{5}(x - 10) \Rightarrow y = -\frac{2}{5}x + 8.$$

(b) The curve meets the x-axis at X: $0 = \dfrac{10}{12-x} - 1 \Rightarrow x = 2$, so $X(2, 0)$. The normal meets the x-axis at N: $0 = -\dfrac{2}{5}x + 8 \Rightarrow x = 20$, so $N(20, 0)$.

$$\text{Shaded area} = \int_{2}^{10} \left(\frac{10}{12-x} - 1\right) dx + \frac{1}{2}(20-10)(4).$$

$$\int_{2}^{10} \left(\frac{10}{12-x} - 1\right) dx = [-10\ln(12-x) - x]_2^{10} = (-10\ln 2 - 10) - (-10\ln 10 - 2) = 10\ln 5 - 8.$$

Adding the triangle $\frac{1}{2}(10)(4) = 20$:

$$\text{Area} = 10\ln 5 - 8 + 20 = 10\ln 5 + 12.$$