Elementary Mathematics

GCE 'O' Levels

Mock Examination · Total: 100 marks · Generated 30/08/2026

Question 1

[5 marks]

PROBABILITY

9 A bag contains 21 marbles, of which 6 are blue and 15 are red. Two marbles are drawn at random, one after the other, without replacement.

(a) Complete the tree diagram.

(b) Find the probability that **(i)** both marbles drawn are blue, **(ii)** at most one of the marbles drawn is red.

Question 2

[4 marks]

RATIO AND PROPORTION

Working at the same rate, 12 identical machines take 8 hours to fill 3000 bottles.

(a) Find the time taken for 15 such machines to fill 3000 bottles.

(b) Find the number of machines needed to fill 5000 bottles in 5 hours.

Question 3

[7 marks]

NUMBERS AND OPERATIONS

Light travels at a speed of 3.0×10^8 metres per second.

(a) A star is 4.1×10^{16} m from Earth. Find the time, in seconds, for light to travel from the star to Earth. Give your answer in standard form, correct to 2 significant figures. [2]

(b) Taking one year to be 3.15×10^7 seconds, find the distance of the star from Earth in light-years, where one light-year is the distance light travels in one year. Give your answer correct to 3 significant figures. [3]

(c) A distant galaxy is 5.0×10^{22} m from Earth. Find how many times further the galaxy is than the star, giving your answer in standard form. [2]

Question 4

[10 marks]

EQUATIONS AND INEQUALITIES

2 Mr Lee paid $\$100$ for his water bill in January when the rate was $\$x$ per cubic metre.

In December, the price of water was increased by 5 cents per cubic metre. By cutting down on usage, Mr Lee still managed to pay $\$100$ for his water bill in December.

(a) Write down an expression, in terms of x, for the amount of water used by Mr Lee in January. [1]

(b) Write down an expression, in terms of x, for the amount of water used by Mr Lee in December. [1]

(c) If the amount of water used in December was $2\ \text{m}^3$ less than the amount used in January, form an equation in x and show that it reduces to $20x^2 + x - 50 = 0$. [3]

(d) Solve the equation and find the price of water per cubic metre in January. [3]

(e) Using your result obtained in part **(d)**, calculate how much water Mr Lee used in December, leaving your answer correct to 2 decimal places. [2]

Question 5

[4 marks]

VECTORS IN TWO DIMENSIONS

14. On the grid, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Mark clearly on the grid: **(i)** the point C such that $\overrightarrow{OC} = 2\mathbf{b} + \mathbf{a}$; **(ii)** the point D such that $\overrightarrow{OD} = \frac{1}{2}\mathbf{b} - 3\mathbf{a}$.

(b) Find $\overrightarrow{CD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Question 6

[11 marks]

FUNCTIONS AND GRAPHS

3 The daily growth rate, h millimetres per day, of Plant A is related to the number of hours of sunlight it receives each day, t, by the formula

$$h = \frac{1}{6}t^2(4 - t).$$

(a) Complete the table of values for $h = \frac{1}{6}t^2(4 - t)$. Values are given to 2 decimal places where appropriate.

t	0.5	1	1.5	2	2.5	3	3.5	4
h		0.5	0.94	1.33	1.56	1.5	1.02	0

(b) On the grid opposite, draw the graph of $h = \frac{1}{6}t^2(4 - t)$ for $0 \le t \le 4$.

(c) Use your graph to find the maximum number of hours of sunlight Plant A can receive to achieve a daily growth rate of 1 millimetre per day.

(d)(i) By drawing a tangent, find the gradient of the curve at $(3, 1.5)$.

(ii) What does the gradient in part (i) tell us about the effect of additional sunlight on the daily growth rate of Plant A at this point? Explain your answer.

The daily growth rate, h millimetres per day, of another Plant B is related to the t hours of sunlight it receives each day by the formula $2h = t$.

(e)(i) On the grid above, draw the graph of $2h = t$ for $0 \le t \le 4$.

(ii) Find an equation in the form $t^3 + pt^2 + qt + r = 0$, where p, q and r are constants to be found, such that the solutions represent the number of hours of sunlight when both Plant A and Plant B have the same daily growth rate.

Question 7

[11 marks]

COORDINATE GEOMETRY

2 The points A, B, C and D have coordinates $(1, 2), (7, 4), (9, 10)$ and $(3, 8)$ respectively.

(a) Show that $ABCD$ is a rhombus. [3]

(b) Find the equation of the diagonal AC and the equation of the diagonal BD, and hence find the coordinates of the point at which the two diagonals meet. [4]

(c) Find the area of $ABCD$. [3]

(d) Find the perimeter of $ABCD$, correct to 3 significant figures. [1]

Question 8

[2 marks]

DATA HANDLING

4 The line graph shows the number of patients in a hospital who are diagnosed with a certain type of blood disorder over the years 2018 to 2022.

(a) Calculate the percentage increase in the number of patients diagnosed with the blood disorder from 2018 to 2019.

(b) State and explain one aspect of the line graph that may be misleading.

Question 9

[3 marks]

ANGLES, TRIANGLES AND POLYGONS

In a regular polygon, the ratio interior angle : exterior angle = $5:1$.

Calculate the number of sides of the polygon.

Question 10

[5 marks]

PYTHAGORAS' THEOREM AND TRIGONOMETRY

24. Roads AB and CD meet at point X at an angle of 58°, with A due north of X. Point Q lies on road CD and is 200 km from point X. At 1040 a truck T is 325 km due north of X and travels at 75 km/h along the straight path TQ to reach Q.

(a) At what time would the truck reach point Q? [3]

(b) Calculate the bearing of Q from T. [2]

Question 11

[3 marks]

RATE AND SPEED

9 On a particular day, the lowest temperature recorded was -5
$,^\circ$ C.

The difference between the highest and lowest temperature recorded that day was 6
$,^\circ$ C.

(a) Find the highest temperature on that day. [1]

(b) The lowest temperature was recorded at 0400 and the highest temperature was recorded at 1200.

The temperature is assumed to increase at a constant rate between 0400 and 1200 that day.

Find the time when the temperature was -1.5
$,^\circ$ C. [2]

Question 12

[9 marks]

SET LANGUAGE AND NOTATION

ξ = {x : x is an integer and $-6 \le x \le 6$}

A = {x : $x^2 \le 9$}

B = {x : $2x + 5 > 1$}

C = {x : x is an odd number}

(a) List the elements of $A \cap B$. [2]

(b) List the elements of $A' \cap C$. [2]

(c) Find $n(B \cup C)$. [2]

(d) Explain why C ◻ B. [1]

(e) Rina says that $A \cap B \cap C$ contains exactly two elements. Explain whether Rina is correct. [2]

Question 13

[5 marks]

MATRICES

20 A concert was held over a particular weekend. The matrix M shows the number of tickets sold on Saturday and Sunday respectively, with the rows representing children, adults and senior citizens.

M = \\begin{pmatrix} 84 & 51 \\\\ 135 & 160 \\\\ 72 & 87 \\end{pmatrix}

(a) The concert tickets were priced at \\$6 for a child, \\$15 for an adult and \\$8 for a senior citizen.

Represent this information in a 1
$times$3 matrix P. [1]

(b) Evaluate the matrix $T = PM$. [2]

(c) State what each of the elements in matrix T represents. [1]

(d) The elements of the matrix N, where $N = QM$, represent the number of tickets sold on each day of the concert.

Write down the matrix Q. [1]

Question 14

[7 marks]

MENSURATION

2 A closed container, in the shape of a cone, is partially filled with water to a height of 25 cm as shown in Diagram I. The diameter of the container is 38 cm and the height of the container is 40 cm.

(a) Calculate the amount of water in the container, leaving your answer in litres. [3]

The container is subsequently inverted so that the water flows to the base of the cone as shown in Diagram II.

(b) Calculate the height of the water, h cm, in the cone as shown in Diagram II. [4]

Question 15

[9 marks]

PERCENTAGE

(a) The price of a laptop is $\$1879$ in Singapore. The price of the same laptop in the USA is USD $\$1299$. The exchange rate between Singapore dollars (SGD) and US dollars (USD) is SGD $\$1$ = USD $\$0.71$. Calculate how much cheaper the laptop is in the USA than in Singapore. Leave your answer in SGD.

(b) The price of a camera after a 7% tax refund is $\$925$. Calculate the price of the camera before the tax refund, giving your answer to the nearest dollar.

(c) The table shows the prices of the electronic devices of Brand A.

Type	11-inch	13-inch	14-inch
Tablet Lite	\$879	\$1099	\$1209
Tablet Pro	\$1099	$\$x$	\$1999
Laptop	\$1879	\$2179	\$2329

(i) The price difference of the devices follows a pattern. Calculate the value of x.

(ii) Calculate the percentage increase in the price of a 14-inch laptop from an 11-inch laptop.

(iii) The total revenue made by Brand A in 2021 was $\$1.6$ million, rounded to the nearest dollar. Given that the sales of the 11-inch Tablet Lite contributed 50% of its revenue, calculate the mean number of 11-inch Tablet Lite sold per day in 2021. Give your answer to the nearest whole number.

Question 16

[5 marks]

ALGEBRAIC EXPRESSIONS AND FORMULAE

3. The first three terms in a sequence of numbers, T_1, T_2, T_3, ..., are given below.

$$T_1 = 3^0 + 1 + 2^2 = 6$$

$$T_2 = 3^1 + 4 + 3^2 = 16$$

$$T_3 = 3^2 + 7 + 4^2 = 32$$

(a) Find T_4.

(b) Find an expression, in terms of n, for the nth term, T_n, of the sequence.

(c) Consider the following sequence.

$$S_1 = -T_1, \qquad S_2 = T_2, \qquad S_3 = -T_3$$

Using your answer from **(b)**, find an expression for the nth term, S_n, of the sequence.

Answers

Question 1

(a) First draw: $P(\text{Blue}) = \frac{6}{21} = \frac{2}{7}$, $P(\text{Red}) = \frac{15}{21} = \frac{5}{7}$.

Second draw after Blue: $P(\text{Blue}) = \frac{5}{20} = \frac{1}{4}$, $P(\text{Red}) = \frac{15}{20} = \frac{3}{4}$.

Second draw after Red: $P(\text{Blue}) = \frac{6}{20} = \frac{3}{10}$, $P(\text{Red}) = \frac{14}{20} = \frac{7}{10}$.

(b)(i) $P(\text{both blue}) = \frac{2}{7} \times \frac{1}{4} = \frac{1}{14}$.

(ii) At most one red = $1 - P(\text{both red})$.

$P(\text{both red}) = \frac{15}{21} \times \frac{14}{20} = \frac{5}{7} \times \frac{7}{10} = \frac{1}{2}$.

So $P(\text{at most one red}) = 1 - \frac{1}{2} = \frac{1}{2}$.

Question 2

(a) Work for 3000 bottles = $12 \times 8 = 96$ machine-hours. Time = $\frac{96}{15} = 6.4$ hours.

(b) For 5000 bottles, work = $96 \times \frac{5000}{3000} = 160$ machine-hours. Machines = $\frac{160}{5} = 32$

.

Question 3

(a) Time = $\frac{4.1 \times 10^{16}}{3.0 \times 10^{8}} = 1.367 \times 10^{8} \approx 1.4 \times 10^{8}$ s (2 s.f.).

(b) One light-year = $3.0 \times 10^{8} \times 3.15 \times 10^{7} = 9.45 \times 10^{15}$ m. Distance = $\frac{4.1 \times 10^{16}}{9.45 \times 10^{15}} = 4.339 \approx 4.34$ light-years (3 s.f.).

(c) $\frac{5.0 \times 10^{22}}{4.1 \times 10^{16}} = 1.2195 \times 10^{6} \approx 1.22 \times 10^{6}$ times.

Question 4

(a) Amount of water = $\frac{\text{bill}}{\text{rate}} = \frac{100}{x}$ m^3.

(b) December rate = $(x + 0.05)$ dollars per m^3, so the amount used = $\frac{100}{x + 0.05}$ m^3.

(c) December usage is 2 m^3 less than January usage:

$$\frac{100}{x} - \frac{100}{x + 0.05} = 2$$

Multiply throughout by $x(x + 0.05)$:

$$100(x + 0.05) - 100x = 2x(x + 0.05)$$

$$100x + 5 - 100x = 2x^2 + 0.1x$$

$$5 = 2x^2 + 0.1x$$

Multiply by 10 and rearrange:

$$20x^2 + x - 50 = 0$$

(d) Using the quadratic formula with $a = 20, b = 1, c = -50$:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(20)(-50)}}{2(20)} = \frac{-1 \pm \sqrt{4001}}{40}$$

$$x = \frac{-1 + 63.2456}{40} = 1.5561 \quad \text{or} \quad x = \frac{-1 - 63.2456}{40} = -1.6061$$

So $x = 1.56$ or $x = -1.61$ (3 s.f.). A price cannot be negative, so the price of water in January is $\$1.56$ per m^3.

(e) Using the unrounded value $x = 1.55614$:

$$\text{December usage} = \frac{100}{1.55614 + 0.05} = \frac{100}{1.60614} = 62.2610 \ldots$$

So Mr Lee used 62.26 m^3 of water in December (the mark scheme quotes 62.3 m^3 to 3 s.f.).

Question 5

(a) Plot C at position vector $2\mathbf{b} + \mathbf{a}$ and D at position vector $\frac{1}{2}\mathbf{b} - 3\mathbf{a}$ from O.

(b)

$$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD} = -(2\mathbf{b} + \mathbf{a}) + \ \frac{1}{2}\mathbf{b} - 3\mathbf{a} \ = -4\mathbf{a} - \frac{3}{2}\mathbf{b}.$$

Question 6

(a) For $t = 0.5$: $h = \frac{1}{6}(0.5)^2(4 - 0.5) = \frac{1}{6}(0.25)(3.5) = 0.15$. The completed table is:

t	0.5	1	1.5	2	2.5	3	3.5	4
h	0.15	0.5	0.94	1.33	1.56	1.5	1.02	0

(b) Plot a smooth curve through these points; it rises to a maximum near $t = 2.5$ then falls to 0 at $t = 4$.

(c) Reading where the curve last meets $h = 1$: about 3.5 hours.

(d)(i) The tangent at $(3, 1.5)$ has gradient about -0.5 (accept ± 0.1).

(d)(ii) The gradient is negative, so at this point the daily growth rate is decreasing; additional sunlight reduces Plant A's daily growth rate.

(e)(i) $2h = t$ is the straight line $h = \frac{t}{2}$ through $(0, 0)$ and $(4, 2)$.

(e)(ii) Setting the growth rates equal:

$$\frac{1}{6}t^2(4 - t) = \frac{t}{2}$$

$$t^2(4 - t) = 3t$$

$$t^3 - 4t^2 + 3t = 0.$$

So $p = -4, q = 3, r = 0$.

Question 7

(a)

$$AB^2 = (7-1)^2 + (4-2)^2 = 36 + 4 = 40$$

$$BC^2 = (9-7)^2 + (10-4)^2 = 4 + 36 = 40$$

$$CD^2 = (3-9)^2 + (8-10)^2 = 36 + 4 = 40$$

$$DA^2 = (1-3)^2 + (2-8)^2 = 4 + 36 = 40$$

Each side has length $\sqrt{40} = 2\sqrt{10}$ units. A quadrilateral with four equal sides is a rhombus, so $ABCD$ is a rhombus.

(b) Gradient of $AC = \dfrac{10-2}{9-1} = 1$. Using $A(1, 2)$: $2 = 1 + c \Rightarrow c = 1$, so AC is $y = x + 1$.

Gradient of $BD = \dfrac{8-4}{3-7} = -1$. Using $B(7, 4)$: $4 = -7 + c \Rightarrow c = 11$, so BD is $y = -x + 11$.

At the point of intersection, $x + 1 = -x + 11 \Rightarrow 2x = 10 \Rightarrow x = 5$ and $y = 6$.

The diagonals meet at $(5, 6)$.

(c) $AC = \sqrt{8^2 + 8^2} = \sqrt{128} = 8\sqrt{2}$ units and $BD = \sqrt{(-4)^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$ units.

The diagonals of a rhombus are perpendicular, so

$$\text{Area} = \frac{1}{2} \times 8\sqrt{2} \times 4\sqrt{2} = \frac{1}{2} \times 64 = 32 \text{ units}^2$$

(d) Perimeter = $4 \times 2\sqrt{10} = 8\sqrt{10} = 25.298\ldots = 25.3$ units.

Question 8

(a)

The number of patients rose by 10, from 10 in 2018 to 20 in 2019.

$$\text{Percentage increase} = \frac{10}{10} \times 100\% = 100\%$$

(b)

The scale used on the vertical axis is very small while the scale used on the horizontal axis is very large. The increase in the number of patients therefore looks much smaller and more gradual than it really is: from 2018 to 2019 there was a sharp increase of 100%, but the graph shows what looks like a gentle rise.

Question 9

The interior and exterior angles at a vertex are supplementary, so they add to 180°.

Splitting 180° in the ratio $5:1$ gives

$$\text{exterior angle} = \frac{1}{6} \times 180^\circ = 30^\circ.$$

For a regular polygon with n sides, the exterior angles sum to 360°, so

$$n = \frac{360^\circ}{30^\circ} = 12.$$

The polygon has 12 sides.

Question 10

In triangle TXQ: $TX = 325$ km, $XQ = 200$ km and angle $TXQ = 58^\circ$ (angle between the north line XT and XQ).

(a) By the cosine rule, $TQ^2 = 325^2 + 200^2 - 2(325)(200)\cos 58^\circ = 76735$, so $TQ = 277$ km.

Time = $\dfrac{277}{75} = 3.69$ h = 3 h 42 min. Starting at 1040, the truck arrives at about 1422.

(b) By the sine rule, $\dfrac{\sin(\angle XTQ)}{XQ} = \dfrac{\sin 58^\circ}{TQ}$, so $\sin(\angle XTQ) = \dfrac{200\sin 58^\circ}{277} = 0.612$ and angle $XTQ = 37.8^\circ$.

From T, the point X is due south (bearing 180°), and Q lies to the east of TX, so the bearing of Q from $T = 180^\circ - 37.8^\circ = 142.2^\circ$.

Question 11

(a) Highest temperature

$$= -5 + 6 = 1$$

,$^{\circ}$ C.

(b) From 0400 to 1200 is 8 hours, over which the temperature rises by 6
,$^{\circ}$ C, so the rate is

$$\frac{6}{8} = 0.75$$

,$^{\circ}$ C per hour.

Rise needed to reach -1.5
,$^{\circ}$ C:

$$-1.5 - (-5) = 3.5$$

,$^{\circ}$ C.

Time taken:

$$\frac{3.5}{0.75} = 4\tfrac{2}{3} \text{ hours} = 4 \text{ h } 40 \text{ min.}$$

Adding to 0400 gives the time **0840**.

Question 12

(a) From $x^2 \leq 9$, $A = \{-3, -2, -1, 0, 1, 2, 3\}$, and $2x + 5 > 1$ gives $x > -2$, so $B = \{-1, 0, 1, 2, 3, 4, 5, 6\}$. Hence

$$A \cap B = \{-1, 0, 1, 2, 3\}$$

(b) $A^{'} = \{-6, -5, -4, 4, 5, 6\}$ and $C = \{-5, -3, -1, 1, 3, 5\}$, so

$$A^{'} \cap C = \{-5, 5\}$$

(c) B has 8 elements, and the only elements of C not already in B are -5 and -3, so $B \cup C = \{-5, -3, -1, 0, 1, 2, 3, 4, 5, 6\}$ and

$$n(B \cup C) = 10$$

(d) $-5 \in C$, but $2(-5) + 5 = -5$, which is not greater than 1, so $-5 \notin B$. An element of C lies outside B, hence $C \not\subseteq B$.

(e) Rina is not correct. Using $A \cap B = \{-1, 0, 1, 2, 3\}$ from part **(a)** and keeping only the odd elements,

$$A \cap B \cap C = \{-1, 1, 3\}$$

which contains three elements, not two.

Question 13

(a) P = \\begin{pmatrix} 6 & 15 & 8 \\end{pmatrix}

(b) T = PM = \\begin{pmatrix} 6 & 15 & 8 \\end{pmatrix}\\begin{pmatrix} 84 & 51 \\\\ 135 & 160 \\\\ 72 & 87 \\end{pmatrix}

Saturday element = $6(84) + 15(135) + 8(72) = 504 + 2025 + 576 = 3105$

Sunday element = $6(51) + 15(160) + 8(87) = 306 + 2400 + 696 = 3402$

T = \\begin{pmatrix} 3105 & 3402 \\end{pmatrix}

(c) The elements of T represent the total amount of money, in dollars, collected from ticket sales on Saturday and on Sunday respectively.

(d) To total the tickets of all three categories for each day, Q must be a 1 $times 3$ matrix of ones:

Q = \\begin{pmatrix} 1 & 1 & 1 \\end{pmatrix}

Question 14

(a) The base radius of the container is $\dfrac{38}{2} = 19$ cm.

The water forms a cone similar to the container, of height 25 cm and radius r:

$$\frac{r}{19} = \frac{25}{40} \Rightarrow r = \frac{25}{40} \times 19 = 11.875 \text{ cm}$$

$$V = \frac{1}{3}\pi(11.875)^2(25) = 3691.78 \text{ cm}^3$$

$$V = 3.69 \text{ litres (3 s.f.)}$$

(b) Volume of the whole container:

$$V_{\text{cone}} = \frac{1}{3}\pi(19)^2(40) = 15121.6 \text{ cm}^3$$

After inverting, the empty space at the top is a cone similar to the container. Let its height be h_1.

$$\left(\frac{h_1}{40}\right)^3 = \frac{\frac{1}{3}\pi(19)^2(40) - \frac{1}{3}\pi(11.875)^2(25)}{\frac{1}{3}\pi(19)^2(40)}$$

$$h_1 = 40\sqrt[3]{\frac{(19)^2(40) - (11.875)^2(25)}{(19)^2(40)}} = 36.437 \text{ cm}$$

Height of water:

$$h = 40 - 36.437 = 3.56 \text{ cm (3 s.f.)}$$

Question 15

(a) In SGD, the USA price is $\frac{1299}{0.71}$ = $1829.58. It is cheaper by $1879 - 1829.58 =$ $49.42.

(b) After a 7% refund the price is 93% of the original: $\frac{100}{93} \times 925 = \$994.62 \approx \$995$.

(c)(i) For each row the second price gap is half the first. For the Tablet Pro, $1999 - 1099 = 900 = 1.5d$ where d is the first gap, so $d = 600$ and $x = 1099 + 600 = 1699$.

(c)(ii) Percentage increase = $\frac{2329 - 1879}{1879} \times 100\% = \frac{450}{1879} \times 100\% = 23.9\%$ (to 3 s.f.).

(c)(iii) Revenue from 11-inch Tablet Lite = $50\% \times \$1\,600\,000 = \$800\,000$. Number sold = $\frac{800\,000}{879} = 910.1$. Mean per day = $\frac{910.1}{365} = 2.49 \approx 2$.

Question 16

(a) Following the pattern, the three parts are 3^{n-1}, $3n - 2$ and $(n + 1)^2$:

$$T_4 = 3^3 + 10 + 5^2 = 27 + 10 + 25 = 62$$

(b)

$$T_n = 3^{n-1} + (3n - 2) + (n + 1)^2$$

$$= 3^{n-1} + 3n - 2 + n^2 + 2n + 1$$

$$= 3^{n-1} + n^2 + 5n - 1$$

(c) The signs alternate, starting negative, so

$$S_n = (-1)^n T_n = (-1)^n \; 3^{n-1} + n^2 + 5n - 1$$

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